# Epimorphisms and Acyclic Types in Univalent Mathematics

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# Starting question

- Exercise in category theory:
  *The epimorphisms of sets are precisely the surjections.*
- **Question**:
  *What are the epimorphisms of types?*
- We answer this question in homotopy type theory (HoTT), where we have higher types.

# Motivation for studying epimorphisms

- Epimorphisms are useful because

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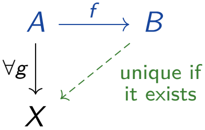

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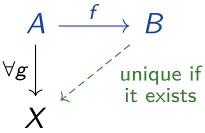



- We show that epis of types are closely related to acyclic types.

  Classically, acyclic spaces are used in algebraic topology in
  - Quillen's plus construction,
  - the Kan–Thurston theorem, and
  - the Barratt–Priddy(–Quillen) theorem.

  So this leads to interesting synthetic homotopy theory!

# Homotopy type theory (HoTT)

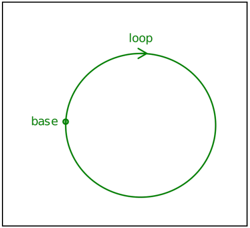
- In HoTT, we think of types as spaces.
- If we have a type $A$ with points $a, b : A$, then we may have identifications $p, q : a =_A b$ and higher identifications $\alpha, \beta : p =_{a =_A b} q$, etc.
- A type is a set or 0-type if there are no higher identifications. E.g. $\mathbb{N}$, $\mathbb{N} \to \mathbf{2}$, $\mathbb{N} \to \mathbb{N}$, etc. are all 0-types.

# Higher types

- The circle $\mathbb{S}^1$

  **Higher inductive type**

  $$\mathsf{base} : \mathbb{S}^1$$
  $$\mathsf{loop} : \mathsf{base} = \mathsf{base}$$

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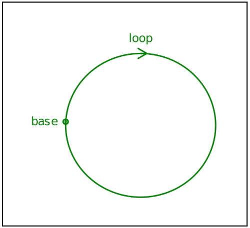


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- Similarly, we get the notion of a $k$-type for $k \geq 0$.
(Actually, $k \geq -2$.)

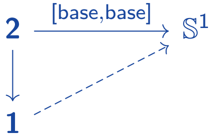
# Synthetic homotopy theory

- Everything we do in HoTT is automatically/necessarily invariant under homotopy.
- This is both a blessing (no need for: "up to...") and a curse as it means that some (point-set based) constructions are not (readily) available in HoTT.
- In practice this means we work with universal properties only.

# Epimorphisms and the circle

- The terminal map $\mathbf{2} \to \mathbf{1}$ is an epi of sets, but *not* of (higher) types!

- Indeed, the type of extensions (dashed) in the diagram




is described as

$$\sum_{x:\mathbb{S}^1} (x = \text{base}) \times (x = \text{base}) \quad \simeq \quad (\text{base} = \text{base}) \quad \simeq \quad \mathbb{Z},$$

which has infinitely many elements.

# Epimorphisms in HoTT

- In 1-category theory, a morphism $f : A \to B$ is an epi(morphism) if for every object $C$ and all morphisms $g, h : B \to C$, we have

$$(g \circ f = h \circ f) \implies (g = h).$$

In other words, $(-) \circ f$ is an injection.

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- We want the epis to be a subtype of the type of functions. That is,

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- <u>Def</u>. A map $f : A \to B$ is an epi if the canonical map

$$(g = h) \longrightarrow (g \circ f = h \circ f)$$

  is an *equivalence* for all types $C$ and all maps $g, h : B \to C$.

  In other words, $(-) \circ f$ is an embedding.

# Suspensions and acyclic types

- Def. The suspension $\Sigma A$ of a type $A$ is the pushout

$$\begin{array}{ccc} A & \longrightarrow & \mathbf{1} \\ \downarrow & & \downarrow S \\ \mathbf{1} & \xrightarrow[N]{} & \Sigma A \end{array}$$

Higher inductive type

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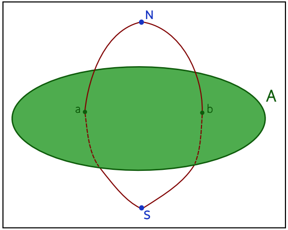


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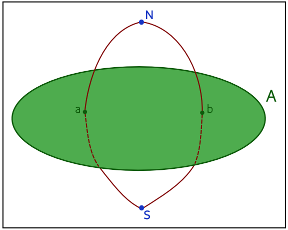


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- Def. A type $A$ is acyclic if $\Sigma A$ is contractible, i.e. $\Sigma A \simeq \mathbf{1}$.

- Ex. The unit type is acyclic. More interesting examples later!

# Characterization of epimorphisms

- Fact A map $f : X \to Y$ is epi w.r.t. *sets* $\iff$ $f$ is surjective.

- Surjectivity means: for every $y : Y$, the fiber of $f$ is inhabited.
  That is, we have an element of the propositional truncation of

$$\mathrm{fib}_f(y) := \sum_{x:X} f(x) = y.$$

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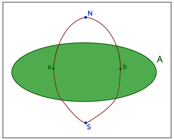
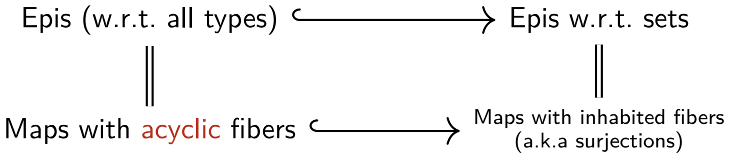
**Theorem**

A map $f : X \to Y$ is epi (w.r.t. all types) $\iff$ all fibers are acyclic.

That is, the suspension of $\mathrm{fib}_f(y)$ is equivalent to $\mathbf{1}$ for all $y : Y$.



# The results so far



# The bigger picture

$$\begin{array}{ccc}
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- Def. A type $A$ is $k$-acyclic if its suspension is $k$-connected, i.e. $\|\Sigma A\|_k \simeq \mathbf{1}$.
- Note: sets are exactly the 0-types and a type is 0-acyclic if and only if it is inhabited, so we recover the results for sets and surjections.

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- Note: sets are exactly the 0-types and a type is 0-acyclic if and only if it is inhabited, so we recover the results for sets and surjections.
- We have nice characterizations of $k$-acyclic types for small $k$:

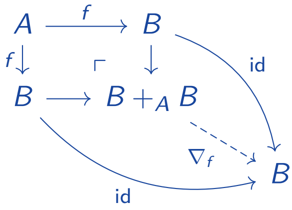
  1-acyclic $\iff$ 0-connected

  2-acyclic $\iff$ 0-connected and perfect fundamental group

  (A group is perfect if it's equal to its commutator subgroup.)

# Proving the characterization of epimorphisms

- Lemma A map $f : A \to B$ is epic if and only if its codiagonal $\nabla_f$ is an equivalence.



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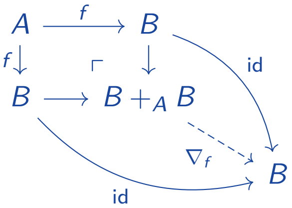


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$$\mathrm{fib}_{\nabla_f}(b) \simeq \Sigma\, \mathrm{fib}_f(b).$$

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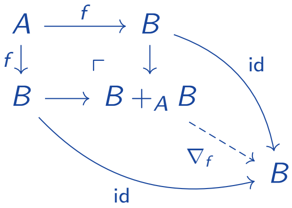


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- Thm. A map is epic if and only if all its fibers are acyclic.

# Towards examples of acyclic types

- For building examples of acyclic types (and hence epis) it helps to be familiar with classifying types/deloopings of groups.
- Buchholtz, Rijke and van Doorn showed that there is a equivalence between the categories of
  - groups with group homomorphisms,
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  - groups with group homomorphisms,
  - 0-connected, pointed 1-types with pointed maps.
- Given a group $G$ we construct a 0-connected 1-type $\mathrm{B}G$ with a point $\mathrm{pt} : \mathrm{B}G$ such that we have an isomorphism of groups

$$\Omega\,\mathrm{B}G :\equiv (\mathrm{pt} = \mathrm{pt}) \cong G.$$

  The group structure on $\Omega\,\mathrm{B}G$ is concatenation of paths.
- We call $\mathrm{B}G$ the classifying type or delooping of $G$.

# No acyclic sets

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- Proof. Let $G$ be the free group on an acyclic set $A$ with inclusion of generators $\eta : A \hookrightarrow G$. If $A$ is acyclic, then $A \to \mathbf{1}$ is an epi, so the constant map

$$\mathrm{B}G \to (A \to \mathrm{B}G)$$

is an embedding. Hence, the constant map $G \to (A \to G)$ is an equivalence. Thus, $\eta$ is constant. But it is also an embedding, so $A$ must be a subsingleton. Finally, $A$ is also inhabited, because it is 0-acyclic. □

# Hatcher's 2-dimensional complex

- Hatcher's 2-dimensional complex is an example of a nontrivial acyclic space.
- We import Hatcher's 2-dimensional complex as a HIT $X$ with constructors:

$$\mathrm{pt} : X, \quad a, b : \Omega X, \quad r : a^5 = b^3, \quad s : b^3 = (ab)^2$$

- Prop. The type $X$ has a 0-connected map to $\mathrm{B}A_5$, the classifying type of the alternating group $A_5$.
  So $X$ is nontrivial.

# Towards acyclicity of Hatcher's complex

- Def. A Hatcher structure on a pointed type $A$ is given by identifications

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- Lemma Every loop space, pointed at refl, has a unique Hatcher structure.

- Proof. The type of Hatcher structures a loop space $\Omega A$ is

$$\sum_{a,b:\Omega^2 A} \left(a^5 = b^3\right) \times \left(b^3 = (ab)^2\right).$$

By Eckmann–Hilton, we have $ab = ba$, so the last component is equivalent to $b = a^2$, and can be contracted away to obtain: $\sum_{a:\Omega^2 A}(a^5 = a^6)$. But, cancelling $a^5$, this is equivalent to the contractible type $\sum_{a:\Omega^2 A}(a = \mathsf{refl})$. □

# Acyclicity of Hatcher's complex

- Prop. The type $X$ is acyclic.
- Proof. For all pointed types $Y$, we have:

$$\begin{aligned}(\Sigma X \to_{\mathsf{pt}} Y) &\simeq (X \to_{\mathsf{pt}} \Omega Y) \\ &\simeq \text{Hatcher-structure}(\Omega Y) \\ &\simeq \mathbf{1}.\end{aligned}$$

Thus, $\Sigma X$ has the universal property of the unit type and hence must be contractible. □

# Higman's type

- Higman's group is given by the presentation

  $$\mathsf{H} \coloneqq \langle a, b, c, d \mid a = [d, a], b = [a, b], c = [b, c], d = [c, d] \rangle,$$

  where $[x, y]$ is the commutator $[x, y] = xyx^{-1}y^{-1}$.

- Its classifying type $\mathsf{BH}$ is easily described as a HIT with a point constructor $\mathsf{pt} : \mathsf{BH}$, four path constructors $a, b, c, d : \Omega\,\mathsf{BH}$ and four 2-cell constructors for the relations.

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- Similar to Hatcher's example, Eckmann–Hilton implies that $\mathsf{BH}$ is acyclic as the commutators become trivial in higher loop types.

- The group can be shown to be nontrivial, but the classical proof requires combinatorial group theory.

  For $\leq 3$ generators and relations the presentation yields the trivial group!

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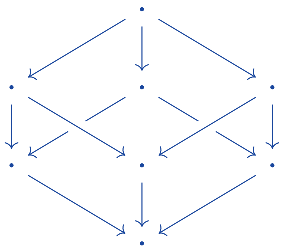
    In a commutative cube whose bottom square is a pushout and whose back sides are pullbacks,



the front sides are pullbacks $\iff$ the top square is a pushout.

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  - Thm. (Wärn) Given 0-truncated maps of 1-types $A \leftarrow R \rightarrow B$, the pushout $A +_R B$ is again a 1-type and the inclusion maps are 0-truncated.

    Such 0-truncated maps give inclusions on loop spaces/groups.

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- We can (re)construct BH as a series of such pushout squares.
- It also follows that BH is a 1-type: no need to truncate!

# Nontriviality of Higman's type: proof sketch

- We re-express BH as an iterated pushout:

$$
\begin{array}{ccc}
\mathrm{B}\langle b\rangle & \longrightarrow & \mathrm{B}\langle b, c\rangle \\
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  Here, each type is the HIT that uses only the constructors of BH that involve the mentioned generators.

  In particular, $\mathrm{B}\langle b\rangle \simeq \mathbb{S}^1$ and $\mathrm{B}\langle a, c\rangle \simeq \mathbb{S}^1 \vee \mathbb{S}^1$.

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- By Wärn's theorem, if the span maps are 0-truncated, then each generator $a, b, c, d$ has infinite order in BH which must also be a 1-type.

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- The type $\mathrm{B}\langle a, b\rangle$ classifies the Baumslag–Solitar group

$$\mathrm{BS}(1,2) = \langle a, b \mid aba^{-1} = b^2\rangle,$$

and is a so-called HNN-extension.
On classifying types this translates to a coequalizer (bottom left), or equivalently, a pushout square (bottom right):

$$\mathbb{S}^1 \underset{b^2}{\overset{b}{\rightrightarrows}} \mathrm{B}\langle b\rangle \longrightarrow \mathrm{B}\langle a, b\rangle \qquad \begin{array}{ccc} \mathbb{S}^1 + \mathbb{S}^1 & \xrightarrow{\nabla} & \mathbb{S}^1 \\ {\scriptstyle [1,2]}\downarrow & \ulcorner & \downarrow \\ \mathrm{B}\langle b\rangle & \longrightarrow & \mathrm{B}\langle a, b\rangle \end{array}$$

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- We apply Wärn's theorem to get that the bottom map is 0-truncated. The other span map can be checked directly.

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- Consider the other pushout square

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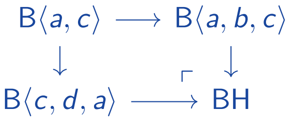
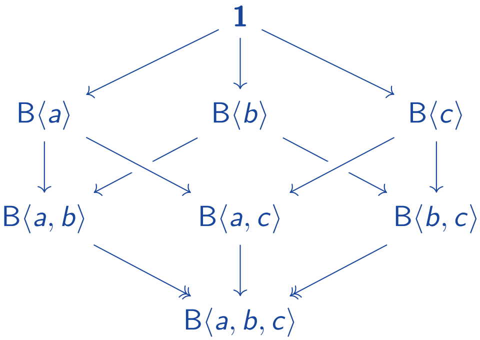



- We apply descent in the commutative cube




to get that the front sides are pullbacks. Since the side maps are 0-truncated, the front map is as well.

# Higman's type

In summary,

**Theorem**

*Higman's type* BH *is an acyclic 1-type in which all four generators have infinite order.*

# Summary

In the presence of higher types, the notion of epimorphism

- becomes quite strong,
- coincides with the notion of an acyclic map, and
- is interesting from the p.o.v. of synthetic homotopy theory.

# Additional and future work

- Details and further results (e.g. closure properties) in:

  Epimorphisms and Acyclic Types in Univalent Mathematics
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- Do the acyclic maps form an accessible modality?

- Use the theory of binate groups to prove acyclicity of some infinitely presented groups?

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