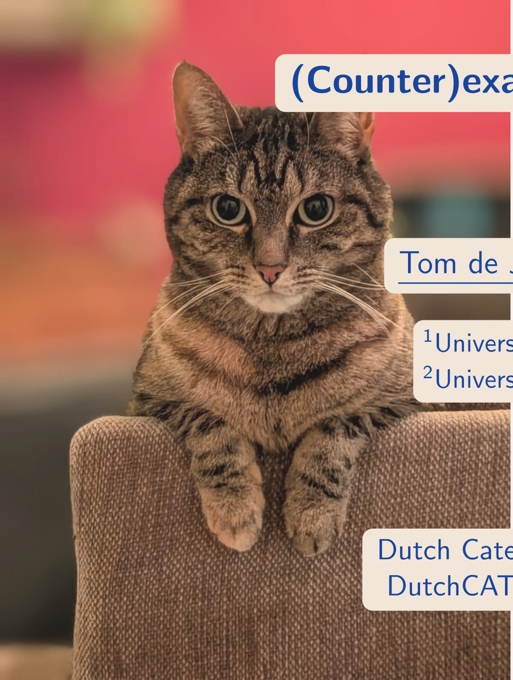

(Counter)examples of injective types

Tom de Jong[1], Martín Escardó[2]

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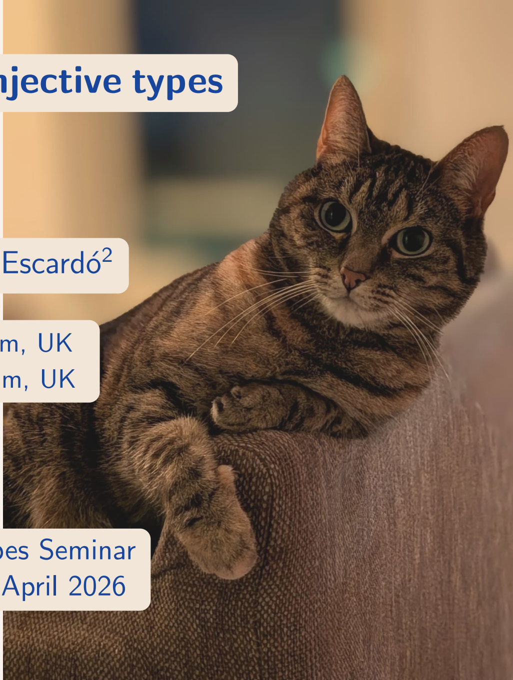

Dutch Categories And Types Seminar
DutchCATS — Delft, 10 April 2026

Motivation

- We work in univalent foundations a.k.a. homotopy type theory (HoTT).
- Injective types were used by Escardó to construct infinite searchable types, see his *TYPES 2019* abstract, but the topic has a rich theory of its own.

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  Use of injectivity: Given a sequence $(X_n)_{n:\mathbb{N}}$ of searchable types, we use the injectivity of the universe to construct a new (searchable) type X_∞ by extending along $\mathbb{N} \hookrightarrow \mathbb{N}_\infty$ and evaluating at $\infty : \mathbb{N}_\infty$.

$$\begin{tikzcd} \mathbb{N} \arrow[rr, hook] \arrow[dr, "X"'] & & \mathbb{N}_\infty \arrow[dl, dashed] \\ & \mathcal{U} & \end{tikzcd}$$

  Can take $\mathbb{N}_\infty :\equiv \Sigma(\alpha : \mathbb{N} \to \mathbf{2}), \Pi(n : \mathbb{N}), \alpha_n \geq \alpha_{n+1}$ with $\infty :\equiv 111\ldots$.

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- In this talk, we present new examples and counterexamples of injective types.

Injective types

- Def. A type D is (algebraically) injective if for every *embedding* $j : X \hookrightarrow Y$, any map $f : X \to D$ into D has a designated extension f/j.

$$\begin{tikzcd}
X \arrow[rr, hook, "j"] \arrow[dr, "f"'] & & Y \arrow[dl, dashed, "f/j"] \\
& D &
\end{tikzcd}
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- The notion of injectivity is sensitive to universe levels, so we really study $\mathcal{U}, \mathcal{V}$-injective types where $X : \mathcal{U}$ and $Y : \mathcal{V}$, but we largely ignore this in this talk.

Injective types can extend partial elements

- Let D be an injective type and P an arbitrary proposition. Suppose we are given a partial element $f : P \to D$ of D.
  By injectivity, we can extend it to a total element $d : D$, as in

$$\begin{tikzcd} P \arrow[rr, hook] \arrow[dr, "f"'] & & \mathbf{1} \arrow[dl, dashed, "d"] \\ & D & \end{tikzcd}$$

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- Types that can extend partial elements to total elements are called flabby.

- Clearly, injective types are flabby. But the converse holds too!

  In short, extend $f : X \to D$ along an embedding $j : X \hookrightarrow Y$ by considering the propositional(!) fibers $j^{-1}\, y :\equiv \Sigma x : X, j\, x = y$ and $f \circ \mathrm{pr}_1 : j^{-1}\, y \to D$.

Examples of injective types

- Any univalent universe $\mathcal{U}$
- The type $\Omega_{\mathcal{U}}$ of propositions in a universe $\mathcal{U}$
- The type $\mathcal{L}\,X :\equiv \Sigma(P : \Omega\mathcal{U}), (P \to X)$ of partial elements of a type $X : \mathcal{U}$
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New examples

- The type of iterative (multi)sets in $\mathcal{U}$
- The types of small ∞-magmas, monoids and groups
- The underlying set of any sup-complete poset, or more generally, of any pointed dcpo

Injective dependent sums

- The injectivity of ∞-magmas, monoids and groups, follows from the injectivity of univalent universes via sufficient criterion for the injectivity of Σ-types.

  The criterion is rather abstract, but importantly, **easy to apply in practice**.

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- Ex. The injectivity of $\Omega_{\mathcal{U}}$ follows by taking $P :=$ is-prop and the retraction given by the propositional truncation.
  This generalizes to any **reflective subuniverse**.

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  (Without propositional resizing, carriers of nontrivial pointed dcpos are large.)

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- Thm. If there is a $\mathcal{U},\mathcal{U}$-injective type in $\mathcal{U}$ with two distinct points, then the type $\Omega_{\neg\neg} :\equiv \Sigma(P : \Omega_{\mathcal{U}}) \times (\neg\neg P \to P)$ of $\neg\neg$-stable propositions in $\mathcal{U}$, whose native universe is $\mathcal{U}^+$, is equivalent to a type in $\mathcal{U}$.

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- The conclusion of the theorem, the resizing of $\Omega_{\neg\neg}$, is **not provable** in univalent foundations. This follows from a proof-theoretic argument due to Andrew Swan.

- This theorem is comparable to a result of Aczel, van den Berg, Granström & Schuster: in the predicative set theory **CZF** it is consistent that the only injective *sets* (as opposed to *classes*) are singletons.

Towards counterexamples of injective types

- With excluded middle, the injective types are precisely the pointed types.
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- But there are plenty of examples of types that cannot be shown to be injective in constructive mathematics, because their injectivity implies a **constructive taboo**: a statement that is not constructively provable and is false in some models.

- The relevant taboo in this case is weak excluded middle: for any proposition P, either $\neg P$ or $\neg\neg P$ holds.
  This is is equivalent to De Morgan's law.

Counterexamples of injective types I

If any of the following types is injective, then weak excluded middle holds.

- The type of booleans $\mathbf{2} :\equiv \mathbf{1} + \mathbf{1}$.
- The simple types, obtained from $\mathbb{N}$ by iterating function types.
- The type of Dedekind reals.
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- The type of Dedekind reals.
- The type of conatural numbers $\mathbb{N}_\infty$.
- More generally, any type with an apartness relation and two points apart.
  Recall: $\boxed{\text{apartness relation}} \approx \boxed{\text{positive (constructive) strengthening of } \neq}$.

Non-properties of injective types

- Thus, injective types, including the universe and the type of propositions, cannot have nontrivial apartness relations.

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- Rice-like theorem: Injective types have no non-trivial decidable properties.

  Thm. If an injective type has a **decomposition**, then weak excluded middle holds.

  A *decomposition* of a type X is defined to be a function $f : X \to \mathbf{2}$ such that we have $x_0 : X$ and $x_1 : X$ with $f\, x_0 = 0$ and $f\, x_1 = 1$.

Counterexamples of injective types II

- While the type $\Sigma(X : \mathcal{U}), X$ of **pointed** types and the type $\Sigma(X : \mathcal{U}), \neg\neg X$ of **non-empty** types are both injective, the type of **inhabited** types need not be.

  Prop. The type $\Sigma(X : \mathcal{U}), \|X\|$ of inhabited types is injective if and only if all propositions are *projective* (a weak choice principle).

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  Prop. If the type $\mathbb{R}\mathrm{P}^\infty :\equiv \Sigma(X : \mathcal{U}_0), \|X \simeq \mathbf{2}\|$ of two-element types (a.k.a. infinite real projective space) is injective, then Fourman–Ščedrov's *world's simplest axiom of choice (WSAC)* holds.

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  WSAC: "choosing an element of a two-element set at most one (subsingleton-many) time(s)".

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  WSAC: "choosing an element of a two-element set at most one (subsingleton-many) time(s)".

- Note that we have:

$$\text{excluded middle} \implies \text{all propositions are projective} \implies \text{WSAC}.$$

Conclusion & future work

- There are plenty of (counter)examples of injective types in univalent foundations!
- Our results on injectives generalize specific observations, e.g. the universe cannot have a nontrivial apartness, ordinals have no non-trivial decidable properties.

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Thank you!

A peak at the Agda formalization

```agda
module Examples-4-13-a where
 open import InjectiveTypes.MathematicalStructures ua

 [1] : ainjective-type (Pointed-type 𝓤) 𝓤 𝓤
 [1] = ainjectivity-of-type-of-pointed-types

 [2] : ainjective-type (∞-Magma 𝓤) 𝓤 𝓤
 [2] = ainjectivity-of-∞-Magma

 [3] : ainjective-type (∞-Magma∙ 𝓤) 𝓤 𝓤
 [3] = ainjectivity-of-∞-Magma∙

 [4] : ainjective-type (monoid.Monoid {𝓤}) 𝓤 𝓤
 [4] = ainjectivity-of-Monoid

 [5] : ainjective-type (group.Group {𝓤}) 𝓤 𝓤
 [5] = ainjectivity-of-Group
```

```agda
Counterexample-7-1 : ainjective-type 𝟚 𝓤 𝓤 ↔ typal-WEM 𝓤
Counterexample-7-1 = 𝟚-ainjective-gives-WEM , WEM-gives-𝟚-ainjective

Lemma-7-2 : WLPO ↔ (Σ f ꞉ (ℕ∞ → ℕ∞) , ((n : ℕ) → f (ι n) ＝ ι 0) × (f ∞ ＝ ι 1))
Lemma-7-2 = WLPO-is-discontinuous' ,
            (λ (f , p) → basic-discontinuity-taboo' f p)

Counterexample-7-3-1 : ainjective-type ℕ∞ 𝓤₀ 𝓤₀ → WLPO
Counterexample-7-3-1 = ℕ∞-injective-gives-WLPO

Counterexample-7-3-2 : ainjective-type ℕ∞ 𝓤 𝓥 → typal-WEM 𝓤
Counterexample-7-3-2 = ℕ∞-injective-gives-WEM

Counterexample-7-4-1 : ainjective-type ℝ 𝓤₁ 𝓤₁
                     → Σ H ꞉ (ℝ → ℝ) ,
                          ((x : ℝ) → (x < 0ℝ → H x ＝ 0ℝ)
                                   × (x ≥ 0ℝ → H x ＝ 1ℝ))
Counterexample-7-4-1 = ℝ-ainjective-gives-Heaviside-function

Counterexample-7-4-2 : ainjective-type ℝ 𝓤 𝓥 → typal-WEM 𝓤
Counterexample-7-4-2 = ℝ-ainjective-gives-WEM
```

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